DRAFT SYLLABUS FOR B.A./B. Sc. (MATHEMATICS), June, 2019

Course Structure: Semester-wise distribution.

First Semester (Total marks: General –100)
GHS 11 : Algebra-I & Calculus – I (100 marks, 80 lectures)

Second Semester (Total marks: General – 100)
GHS 21 : Geometry & Vector Calculus (100 marks, 80 lectures)

Third Semester (Total marks: General – 100)
GHS 31: Statics & Calculus – II (100 marks, 80 lectures)

Fourth Semester (Total marks: General – 100)
GHS 41: Algebra II & Dynamics (100 marks, 80 lectures)

Fifth Semester (Total marks: Honours - 200)
H 51 : Elementary Number Theory (40 marks, 32 lectures)
H 52 : Advanced Calculus I (60 marks, 48 lectures)
H 53 : Differential Equations (40 marks, 32 lectures)
H 54 : Advanced Dynamics . (60 marks, 48 lectures)

Sixth Semester (Total marks: Honours - 200)
H 61 : Advanced Calculus II (40 marks, 32 lectures)
H 62 : Advanced Algebra (60 marks, 48 lectures)
HOP X: Optional paper (100 marks, 80 lectures)

Optional Papers : Any one of the following (100 marks, 80 lectures)
HOP 1 : Computer programming in C & Computer Oriented Numerical Analysis
HOP 2 : Operations Research
HOP 3 : Fluid Mechanics
HOP 4 : Complex Function Theory
HOP 5 : Discrete Mathematics
HOP 6 : Probability Theory
HOP 7 : Elementary Differential Geometry

(Abbreviation: GHS = General and Honours, H = Honours,
HOP X = Honours Optional Paper No. X)

Notes:
1. The distribution for marks for each paper shall be 25% for Internal assessment and 75% for External assessment.
2. Each question paper should contain the paper name and the corresponding paper number as mentioned in the syllabus.

GHS 11
ALGEBRA-I & CALCULUS – I

(Number of Teaching hours: 80; Time:3 hrs; Marks: 100)
(To answer five questions, choosing one out of two questions from each unit)

SECTION – A (Algebra –I, 40 marks)

UNIT I : Brief review of sets, subsets and equality of two sets; relation on a set: reflexive, symmetric, anti-symmetric, transitive; examples from geometry and number systems; equivalence relation and equivalence classes; partitions.
Brief review of functions/mappings, inclusion maps, restriction of a map, composition of maps, associativity, onto, one-one, bijective maps; inverse images of sets, inverse of a bijective map; finite and infinite sets; proof of "If A is a finite set f:A–> A is one one if and only if f is onto"; examples where this assertion does not hold. Graph of a function: real valued functions such as polynomials, rational functions, logarithmic functions, exponential functions and hyperbolic functions. Limits, ε-δ definition, standard theorems on limits, standard limits; continuity: Intuitive idea, ε-δ definition, theorems on sum, differences, product, quotient and composites of continuous functions; continuity of functions.

UNIT II : A brief review of m × n matrix over R/C as a rectangular array of numbers (motivation through systems of linear equations); transpose, conjugate transpose; definition of inverse of a matrix; special type of matrices: diagonal , scalar, upper/lower triangular, nilpotent, idempotent, symmetric, skew symmetric, Hermitian, skew Hermitian matrices; trace of a square matrix; row vectors and column vectors of a matrix; row rank/column rank of an m × n matrix (in terms of linear independence of row/column vectors of the matrix); adjoint of a matrix; inverse in terms of adjoints; determinantal rank of matrix; equality of rank and determinantal rank; Elementary operations; elementary matrices; row/column reduced echelon form of a matrix; determination of the inverse of a matrix by elementary operations; theorem on the equality of row-rank and column-rank; rank of a matrix; determination of the rank by elementary operations; systems of linear equations: homogeneous and non-homogeneous.

SECTION – B (Calculus – I, 60 marks)

UNIT III : Properties of continuous functions defined on closed and bounded intervals, (statements with illustrations only for the following) boundedness, intermediate value theorem, uniform continuity; derivatives of real valued functions on intervals : definition; derivative as a rate measurer, derivative as the gradient of tangent (geometrical interpretation only); theorems on sum, difference, product, quotient and composite of differentiable functions; review of methods of differentiation; successive differentiation; Leibniz's theorem; l'Hôpital's rule (statements only with applications).

UNIT IV : Anti-derivative : review of the standard methods; integration by parts and by partial fractions; integral of a continuous function as the limit of Riemann sum (including sums arising out of unequal distribution of interval); examples of evaluation of integrals from the definition. Definite integrals, fundamental theorem of integral calculus and differentiability of integrals of continuous functions(statements with illustrations only) properties of definite integral, evaluation of integrals using these properties; reduction formulas for $\int sin^n x dx$, $\int cos^n x dx$, $\int tan^n x dx$, $\int e^{ax} sin(mx) dx$, $\int e^{ax} x^n dx$, $\int e^{ax} (logx)^n dx$, $\int sin^n x cos^m x dx$ and their combinations; improper integrals, convergence and evaluation from definition.

UNIT V : Brief review of first order first degree equations; Bernoulli's equation; exact equations; reduction to exact form by integrating factors; differential equations of first order but higher degrees;

Clairut's equation and singular solution; geometrical interpretation applications of first order differential equations to geometric and physical problems (simple cases only) including orthogonal trajectories, introduction of second order homogeneous differential equations with constant coefficients.

Text Books:

1. Bhattacharya, P. B., Jain, S. K., and Nagpaul, S. R. : *Basic Abstract Algebra*, Cambridge University Press, 2003.
2. Maity, K. C. and Ghosh, R. K. : *An introduction to Analysis: Differential Calculus: Part I*, New Cental Book Agency Pvt Ltd., 2011.
3. Maity, K. C. and Ghosh, R. K. : *An Introduction to Analysis: Integral Calculus*, New Central Book Agency Pvt Ltd., 2013.

Reference Books

1. Fraleigh, J. B., *A First Course in Abstract Algebra*, Pearson Education India, 2013.
2. Gopala Krishnan, N.S., *University Algebra*, New Age International Pvt. Ltd. Publishers, 2007.
3. Stewart, J., *Essential Calculus: Early Transcendentals*, Cengage India Pvt Ltd, 2017.
4. Saikia, P. K.: *Linear Algebra, Pearson*, Delhi, 2014.
5. Das, B.C. and Mukherjee, B.N., *Differential Calculus* , UN Dhar and Sons Pvt Ltd, 52nd edition, 2012.
6. Das, B.C. and Mukherjee, B.N., *Integral Calculus* , UN Dhar and Sons Pvt Ltd, 52nd edition, 2012.
7. Thomas, G. B., and Finney, R. L.: *Calculus and Analytic Geometry* (9th Edition), Pearson Education India, 2010.

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GIIS 21

Geometry & Vector Calculus

(Number of Teaching hours:80; Time: 3 hrs; Marks:100)
(To answer five questions, choosing one out of two questions from each unit)

UNIT I : Change of axes – invariants; pairs of straight lines; general equation of second degree; the standard form; reduction of the general equation to standard form; conditions for different conics; General conics : equations of tangents, normals, pairs of tangents, chord of contact, chord in terms of middle points, pole, polar, conjugate lines, diameter , asymptotes.

UNIT II : Polar equation, equation of a conic, directrix, chord, tangent and normal; parabola, ellipse, hyperbola; conjugate diameters of ellipse and hyperbola; rectangular hyperbola; conjugate hyperbola.

UNIT III : Space co-ordinates: rectangular, Cartesian, cylindrical, spherical, and polar; equation of planes; angle between two planes; perpendicular distance of a point from a plane; Sphere – plane section and its equation; sphere through a given circle; tangent plane; pole and polar plane; intersection of two spheres; Equation of a cone with a conic as a guiding curve; enveloping cone; mutually perpendicular generators; tangent planes; reciprocal cone; right circular cone; equation of a cylinder with a conic as a guiding curve; right circular cylinder.

UNIT IV : Products (scalar and vector products) of two, three and four vectors – properties, geometrical significance and applications; Vector valued functions (up to 3 variables); derivatives of such a function of a single variable; properties and geometrical applications; arc length, unit tangent vector; curvature, normal vector; derivatives of scalar and vector products;

UNIT V : Velocity and acceleration in Cartesian co-ordinates, radial and transverse accelerations on smooth curves (simple problems only), tangential and normal components of velocity and acceleration. Directional derivatives, gradient of a scalar- valued function, tangent planes; vector fields, curl and divergence of a vector field, Physical and geometrical interpretation and elementary properties.

Text Books:

1. Das, B.: *Analytical Geometry and Vector Analysis*, Orient Book Co., Calcutta, 1998.
2. Narayan S.: *Analytical Solid Geometry*, S. Chand & Co., New Delhi, 2007.
3. Ghosh, R. K. and Maity, K. C.; *Vector Analysis*, New Central Book Agency, 2011.

Reference Book

1. Stewart, J.: *Essential Calculus Early Transcendentals*, Cengage India Pvt Ltd, 2017.
2. Thomas, G. B., and Finney, R. L. : *Calculus and Analytic Geometry* (9th Edition), Pearson Education India, 2010.
3. Narayan, S. and Mittal, P. K.: *A Text Book of Vector Analysis*, S. Chand & Co., New Delhi, 2010.
4. Chorlton, F.: *Text Book of Dynamics*, CBS Publishers and distributors, Delhi, 2004.
5. Chatterjee, D.: *Vector Analysis* (2nd Revised Edition), PHI Learning Pvt Ltd. New Delhi, 2005.

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GHS 31
Statics and Calculus-II

(Number of Teaching hours:80; Time: 3hrs; Marks:100)
(To answer five questions, choosing one out of two questions from each unit)

SECTION-A (Statics, 40 marks)

UNIT I : Composition and resolution of forces; parallelogram of forces, Components and resolved parts, Coplanar forces: Equilibrium of concurrent forces, Triangle of forces, Lami's Theorem and its converse; Parallel forces; Moment of a force; Definition, geometrical representation of Moments, Varignon's Theorem. Couples; definition, equilibrium of Couples, Equivalence of two Couples, Resultant of Couples, Resultant of a couple and a force.

UNIT II : Reduction of coplanar forces, equilibrium of coplanar forces. Friction: laws of statical friction, laws of limiting friction, solution of problems on equilibrium of heavy bodies (such as uniform rods) resting on plane surfaces.
Centre of gravity: centre of gravity of thin uniform rod, uniform lamina, triangular lamina and lamina in the form of a parallelogram and trapezium.

SECTION-B (Calculus-II, 60 marks)

UNIT III : Sequences of real numbers : definitions of bounded sequence, convergent sequence, limit of a sequence, monotonic sequence; examples; proof of the fact that monotonic and bounded sequences are convergent (using completeness of R as an axiom); Cauchy sequence; Cauchy's general principle of convergence;
Infinite series of real numbers: partial sums, convergent series, comparison test, ratio test, Raabe's test, root test; absolute convergence; Leibnitz's theorem for alternating series; power series; radius of convergence (without the notion of limit superior), standard examples of power series.

UNIT IV : Application of differential calculus: Sign of the derivatives of a real valued function of a real variable, vanishing of $f(x)$; Rolle's theorem; geometric interpretation, mean value theorems; applications of the mean value theorems: (i) increasing and decreasing functions, (ii) concavity

upwards and downwards, (iii) points of inflections, multiple roots. Use of differentials in approximation and error estimates; maxima and minima; asymptotes; curvature of plane curves (Cartesian and parametric equations only);
Real-valued functions of two or three variables: limits; continuity; partial derivatives of first and second orders; Schwarz's theorem (statement only); differentials; chain rules; Euler's theorem on homogeneous functions, proof upto three variable case.

UNIT V : Location of roots of f(x), proof of the fundamental theorem of integral calculus; Taylor's and Maclaurin's theorem with Cauchy's form of remainders; Taylor's and Maclaurin's series; expansion of standard functions such as ex, $sin\ x$, $cos\ x$, $log\ (1+x)$, $(1+x)n$. Applications of integral calculus : determination of (i) areas under simple plane curves, (ii) lengths of simple plane curves, (iii) volume and surface areas of solids of revolution in standard cases. Evaluation of line integrals (in a plane); double integrals; change of order of integration; application in determination of area, volume (simple cases only).

Text books:

1. Das, B. C. and Mukherjee, B. N.: *Statics*, U. N. Dhar & Sons Publications, Kolkata, 2002.
2. Maity, K. C. and Ghosh, R. K.: *Differential Calculus*, New Cental Book Agency Pvt Ltd., 2002.
3. Maity, K. C. and Ghosh, R. K.: *Integral Calculus*, New Cental Book Agency Pvt. Ltd., 2002.

Reference Books:

1. Loney, S. L.: *An elementary treatise on the Dynamics of a particle and of rigid bodies*, New Age International Pvt. Ltd, 2016.
2. Stewart, J.: *Essential Calculus Early Transcendentals*, Cengage India Pvt Ltd, 2017.
3. Bernside, W. S., and Panton, A. W.: *Theory of Equations*, *Vol. I*, S. Chand & Co., New Delhi,2000.
4. Thomas, G. B., and Finney, R. L. : *Calculus and Analytic Geometry* (9th Edition), Pearson Education India, 2010.
5. Narayan S.: *A course of Mathematical Analysis*, S.Chand and Co., 2005.
6. Das, B.C. and Mukherjee B.N., *Differential Calculus* , UN Dhar and Sons Publisher, 52nd edition, 2012.

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GHS 41
Algebra II & Dynamics

(Number of Teaching hours:80; Time: 3hrs; Marks:100)
(To answer five questions, choosing one out of two questions from each unit)

SECTION-A (Algebra II, 40 marks)

UNIT I : Binary operations as maps from $A \times A \rightarrow A$; commutative and associative binary operations; identities and inverses (one-sided as well as two sided); examples; groups: definition; examples of groups such as $Z, Q, R, C, Q^*, R^*, C^*, Z_n, S_n, M_2(R), GL_2(R), R^2, R^3, n^{th}$ roots of unity etc; laws of indices in both additive and multiplicative notation,; right and left cancellation laws; uniqueness of identity and inverses; group tables of groups of low order (up to 8); subgroups : examples; cyclic subgroup generated by an element: cyclic group; abelian group; subgroups of cyclic groups; determination of all subgroups of Z.
Order of an element; examples of elements of finite order, of infinite order; coset with respect to a subgroup in a group; cosets as equivalence classes; Z_n as cosets; Lagrange's theorem and its applications; groups of prime order; Fermat's (little) theorem; Euler's generalizations; application to simple number theoretic problems.

UNIT II : Polynomials over Z / Q / R / C ; addition and multiplication; degree of a polynomial; degree of sum and product of polynomial; the division algorithm; remainder theorem; gcd; the Euclidean algorithm; irreducible polynomials; application of Eisenstein's theorem; unique factorization theorem; roots of a polynomial; factor theorem; fundamental theorem of algebra (statement only); its failure for polynomials over Z / Q / R ; simple tests of irreducibility of polynomials with rational/ integer coefficients; detailed study of roots of a polynomial with real coefficients: immediate consequence of the fundamental theorem of algebra; multiple roots, common roots, complex roots, surd roots; Relation between roots and coefficients of a polynomial; symmetric function of the roots of a polynomial equation with special reference to cubic and biquadratic equations; transformation of equation; Descartes' rule of signs – simple applications; location of roots using Rolle's theorem; solution of a cubic equation by Cardan's method; De Moivre's theorem – application to solution of equation.

SECTION-B (Dynamics, 60 marks)

UNIT III : Rectilinear motion with variable Laws of forces; Force of repulsion varying as displacement, Motion under inverse square Law, Motion of a particle attracted towards the centre of the Earth. Motion under other laws of forces, simple harmonic motion; velocity and acceleration, amplitude, time-period; collision of elastic bodies; direct and oblique impact, loss of energy due to collision, impulsive action between colliding spheres.

UNIT IV : Projectiles; horizontal Range, time of flight, greatest height , position and velocity at any time, path of a projectile is a parabola.
Rectilinear motion in resisting media on a horizontal plane where resistance varies as (i) velocity, (ii) square of velocity, (iii) displacement; vertical motion under gravity where resistance varies as (i) velocity, (ii) square of velocity.

UNIT V : Tangential and normal acceleration on smooth curves, radial & transversal acceleration, motion on a smooth plane curve such as vertical circles and cycloids.
Impulse and Impulsive force, conservation of linear momentum. Work done by a force; work energy equation; potential function; conservative forces.

Text Books:

1. Gopala Krishnan, N.S.: *University Algebra*, New Age International Pvt Ltd Publishers, 2007.
2. Das, B. and Maiti, S.R.: *Higher Algebra* , Ashoke Prakasan ,Kolkata , 2014.
3. Gupta, P. K., and Juneja, R.: *Dynamics*, Ramesh Book Depot, Jaipur, 2003.

Reference Books:

1. Herstein, I.N. : *Topics in Algebra* (Student Edition), Wiley, 2006.
2. Fraleigh, J. B.: *A First Course in Abstract Algebra*, Pearson Education India, 2013
3. Gallian J.A. : *Contemporary Abstract Algebra*, Narosa Book Distributors, 2008.
4. Ray, M.: *A Text Book on Dynamics for B.A./B.Sc. students*, S. Chand Publication, Delhi, 2002.
5. Singh, K. K.: *Text Book of Dynamics*, PHI Learning pvt. Ltd., New Delhi, 2011.
6. Bhattacharya, P. B., Jain, S. K., and Nagpaul, S. R. : *Basic Abstract Algebra*, Cambridge University Press, 2003.

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H 51

Elementary Number Theory

(Number of Teaching hours:32; Time: 2 hours; Marks:40)
(To answer two questions, choosing one out of two questions from each unit)

UNIT I : Divisibility in the set of integers; basic properties; the division algorithm; gcd; elementary properties; the Euclidean algorithm; lcm; primes (in the set of natural numbers); fundamental theorem of arithmetic; Euclid's proof of the infinitude of primes; arbitrary gaps in the distribution of primes; congruences in the set of integers modulo a positive integer; basic properties; complete residue system; reduced residue system; Euler's ϕ-function; Fermat's theorem: Euler's generalization of Fermat's theorem; applications, Wilson's theorem.

Unit II : Solution of congruences; linear congruences; Chinese remainder theorem; consequences of higher degree modulo a prime. Some functions of number theory – greatest integer function; elementary properties; Arithmetic functions, multiplicative functions; functions such as $\phi(n)$, $\mu(n)$, $\sigma(n)$, $\sigma_k(n)$.

Text Books

1. Niven, I., Zuckerman, H.S., and Montogomery, H.L. : *An Introduction to the Theory of Numbers*, Wiley Eastern Ltd., 2000.

Reference Books

1. Telang, S.G. : *Number Theory*, Tata McGraw-Hill, New Delhi, 1996.
2. Burton, David M.: *Elementary Number Theory*, Universal Book Stall, 2001.

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H 52
Advanced Calculus I

(Number of Teaching hours:48; Time: 3hrs; Marks:60)
(To answer three questions, choosing one out of two questions from each unit)

UNIT I : Riemann integral of functions of one variable; Darboux's theorem (statement and application);conditions for integrability; classes of bounded and integrable functions; properties of integrable functions; inequalities for integrals; functions defined by integrals; their continuity and differentiability; Mean value theorems for integrals. Improper integrals; test for convergence when the integrand is non-negative; absolute convergence; tests for absolute and conditional convergence, beta and gamma functions; Abel's theorem, Dirichlet's theorem; Frullani's integral.

UNIT II : Integrals as functions of parameters; continuity, differentiability and integrability of such a function; applications to evaluation of integrals; Improper integrals as functions of a parameter; uniform convergence and tests for uniform convergence; continuity, differentiability and integrability of uniformly convergent improper integrals of continuous functions involving parameters; evaluation of integrals.

UNIT III : Line integral in R^2; Riemann integral of real valued functions of two variables; evaluation of double integrals – change of order of integration; change of variable and simple problems; Green's theorem in R^2, Surface Integral and Stokes Theorem, Volume integral and Gauss's divergence theorem (statements and applications only).

Text Books:

1. Narayan, S.: *A Course of Mathematical Analysis*, S. Chand. Delhi, 2005.
2. Apostol, T.M. : *Mathematical Analysis*, Narosa Book Distributors, 2002.

Reference Books

1. Stewart, J.: *Essential Calculus Early Transcendentals*, Cengage India Pvt Ltd, 2017.

2. Bartle, R. G., and Sherbert, D. R. : *Introduction to Real Analysis* (Wiley India Edition), Wiley & Sons, Inc, 2000.
3. Rudin, W. : *Principles of Mathematical Analysis* (3rd Edition), McGraw Hill Education, 2013.
4. Malik, S. C. and Arora, S. : *Mathematical Analysis*, New Age International Pvt Ltd., 2017.
5. Ghosh, R.K. and Maity, K.C.: *Introduction to Analysis*, New Central Book Agency (P) Ltd, 2002.
6. Raisinghania M.D.: *Elements of Real Analysis*, S. Chand and Co, 2003.

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H 53
Differential Equations

(Number of Teaching hours:32; Time: 2 hours; Marks:40)
(To answer two questions, choosing one out of two questions from each unit)

UNIT 1 : Linear equations of second and third order with constant coefficients, – complementary functions and particular integrals for $x^n e^{ax}, e^{ax}\sin(mx), e^{ax}\cos(mx), x^n\sin(mx), x^n\cos(mx)$; equations of type $a_1x^2y'' + a_2xy^1 + a_3y = f(x)$; Linear differential equations of second order with variable coefficients; homogeneous equations; exact equations; transformation of the equation by changing the dependent variable/the independent variable, Normal form. Method of variation of parameters; simultaneous equations; total differential equation $Pdx + Qdy + Rdz = 0$

UNIT II: Partial differential equation. Formation of equation, solutions of linear equations of first order, Lagrange's methods, Non linear partial differential equations of first order- Standard forms I, II, III & IV; Integral surfaces passing through a given curve, orthogonal surfaces, non linear equations of first order, Charpit's method.

Text Books:

1. Raisinghania , M.D. : *Ordinary and Partial Differential Equations* , S. Chand and Co, 2013.
2. Sneddon, I. N.: *Elements of Partial Differential Equation*, Dover, 2006.

Reference Book:

1. Coddington, Earl A.: *An Introduction to Ordinary Differential Equations*, PHI Learning Pvt. Ltd., New Delhi, 1998.
2. Piaggio, I.: *An Elementary Treatise on Differential Equations and Applications*, G. Bell & Sons, 2000.

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H 54
Advanced Dynamics

(Number of Teaching hours:48; Time: 3 hours; Marks:60)
(To answer three questions, choosing one out of two questions from each unit)

UNIT I : Central forces; central orbits; centre of force, motion of a particle under the action of central forces, motion on a rough cycloid, description of a central conic under a central force; use of reciprocal polar co-ordinate; stability of a nearly circular orbit; use of pedal co-ordinates and pedal equations; apse; apsidal distance; apsidal angle; perihelion and aphelion; Kepler's laws of planetary motion and its deductions.

UNIT II : Moments and products of inertia; moments of inertia of a uniform rod, a rectangular lamina, a parallopiped, a circular ring, a circular disc; theorems of parallel and perpendicular axes about a fixed axis; principal axes (definition only); momental ellipsoid equimomental systems. D'Alembert's Principle, the general equations of motion, motion of the centre of inertia, motion relative to the centre of inertia.

UNIT III : Motion about a fixed axis: moment of the effective forces about the axis of rotation, equation of motion of a rigid body about a fixed axis, expression for kinetic energy of the body and moment of momentum of the rigid body moving about a fixed axis. The compound pendulum: interchangeability of the centre of suspension and centre of oscillation, minimum time of oscillation; motion in two dimensions (only finite forces), friction, kinetic energy of a body moving in two dimensions, moments of momentum about the origin of a body moving in two dimension and problems illustrating the above cases. (varying mass is not included).

Text Book:

1. Loney, S. L.: *An elementary treatise on the Dynamics of a particle and of rigid bodies*, New Age International Pvt Ltd, 2016.

Reference Books:

1. Vasistha, A.R., Agarwal, D.C.: *Dynamics of a particle*, Krishna Prakashan Mandir Publication, 2015.
2. Rohman, M.M : *Rigid Dynamics*, New Central Book Agency, 2011.

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H 61
Advanced Calculus II

(Number of Teaching hours:32; Time: 2hrs; Marks:40)
(To answer five questions, choosing one out of two questions from each unit)

UNIT I : Definition of a metric space with special emphasis on Euclidean space; Basic properties of Euclidean distance function in R^n; neighbourhoods, open sets, closed sets, limit points, interior points in R^n $(n = 1, 2, 3)$; definition of countable and uncountable sets; Bolzano-Weierstrass theorem; Cantor intersection theorem (nested interval) ; Lindelof covering theorem. Compact sets; Heine-Borel theorem; equivalent statements; study of maps from subsets of $R^n \to R^m$ $(m, n = 1, 2, 3)$: continuity, in terms of $\epsilon - \delta$ notation; in terms of inverse images of open and closed sets; elementary properties of continuous functions; continuous functions on compact sets; special cases of continuous real valued functions on closed, bounded intervals of R: bounds.

UNIT II : Intermediate value theorem; uniform continuity; discontinuities of real valued functions; monotonic functions; continuity of the inverse of a strictly monotonic function. R^m-valued functions of two or three variables $(m = 1, 2, 3)$; partial derivatives; directional derivatives; total derivative, Jacobian; change in the order of partial derivatives, Young's Theorem, Schwarz's Theorem and their applications, differentiation of composite functions; chain rule.

Text Books

1. Narayan, S.: *A Course of Mathematical Analysis*, S. Chand. Delhi, 2005.
2. Apostol, T.M. : *Mathematical Analysis*, Narosa Book Distributors, 2002.

Reference Books

1. Stewart, J.: *Essential Calculus Early Transcendentals*, Cengage India Pvt Ltd, 2017.
2. Bartle, R. G., and Sherbert, D. R. : *Introduction to Real Analysis* (Wiley India Edition), Wiley & Sons, Inc, 2000.
3. Rudin, W. : *Principles of Mathematical Analysis* (3rd Edition), McGraw Hill Education, 2013.
4. Malik, S. C. and Arora, S. : *Mathematical Analysis*, New Age International Pvt Ltd., 2017.
5. Ghosh, R.K. and Maity, K.C.: *Introduction to Analysis*, New Central Book Agency (P) Ltd, 2002.
6. Raisinghania M.D.: *Elements of Real Analysis*, S. Chand and Co, 2003.

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H 62
Advanced Algebra

(Number of Teaching hours: 48; Time: 3 hours; Marks:60)
(To answer three questions, choosing one out of two questions from each unit)

UNIT I : Normal subgroups, examples; conditions for a subgroup to be normal; centre of a group; examples; quotient group; homomorphism, kernel and image of a homomorphism, isomorphism of groups – examples and elementary properties. Fundamental theorem of group homomorphism; isomorphism theorems; automorphisms; inner automorphisms; examples; rings (motivation through Z) : definitions and examples of (i) rings with identity, (ii) commutative rings, (iii) rings with and without zero divisors, integral domains, (iv) division rings (v) fields (examples to include Z_p, integers mod p, fields Q, R, C, polynomial rings R[x], matrix rings $M_n(R)$; basic properties of rings; characteristic of rings; finite integral domains with Z_p as an example.

UNIT II : Subrings; ideals: right, left and two-sided; generated by a subset more specifically by a finite number of elements in a commutative ring with 1; principal ideals, example of ideals in Z, Z_n, $M_n(R)$; prime ideals, maximal ideals in a commutative ring with 1, examples; quotient ring with Z_n as an example. Ring homomorphisms; kernels; isomorphisms; homomorphism and isomorphism theorems including the correspondence theorem; determination of ideals in Z_p; divisibility in integral domains (with 1); units, associates, prime elements, irreducible elements, gcd, Euclidean domain, principal ideal domain, unique factorization domains – definitions, examples and basic results.

UNIT III : Vector spaces (motivation through R^2, R^3) – examples, basic properties; subspaces; homomorphisms or linear maps between vector spaces; isomorphisms; standard homomorphism and isomorphism theorems; direct sum (internal and external); linear dependence and independence; basis, dimension; vector space axioms for the set L(V,W) of linear maps; rank and nullity of a linear transformation; "rank+nullity=dimension" theorem; matrix representation of linear transformations; definitions, examples and determination of characteristic roots, characteristic vectors and characteristic polynomial of a linear transformation/matrix (2 × 2,3 × 3 over R). Statement of Cayley-Hamilton theorem.

Text Books:

1. Gopala Krishnan, N.S.: *University Algebra*, New Age International Pvt Ltd Publishers, 2007.
2. Gallian J.A.: *Contemporary Abstract Algebra*, Narosa Book Distributors, 2008.
3. Saikia, P. K.: *Linear Algebra, Pearson*, Delhi, 2014.

Reference Books:

1. Bhattacharya, P. B., Jain, S. K., and Nagpaul, S. R.: *Basic Abstract Algebra*, Cambridge University Press, 2003.
2. Hoffman, K. and Kunze, R.: *Linear Algebra* (2nd edition), PHI Learning Pvt Ltd, 2015.

3. Herstein, I.N.: *Topics in Algebra* (Student Edition), Wiley, 2006.
4. Fraleigh, J. B.: *A First Course in Abstract Algebra*, Pearson Education India, 2013.

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HOP 1
Computer programming in C & Computer oriented Numerical Analysis

Note: This paper consists of two parts A, B. Part A is theory paper consisting of 60 Marks & Part B is Practical Paper consisting of 40 Marks.

Part : A (Theory)

(Number of Teaching hours: 48; Time : 3 hrs; Marks: 60)

(To answer five questions, choosing one out of two questions from each unit)

UNIT I : C fundamentals: The C character set, identifiers and keywords, Data types, constants, variables and arrays, declarations, symbolic constants, Operators (Arithmetic, unary, relational, logical, bitwise, assignment), expressions, statements, C program structure, Need of header files, Process of compiling and running a C program; I/O functions: Header files (stdio.h, conio.h), getch(), getche(), getchar(),putch(), putchar(), scanf(), printf(), gets(), puts(), clrscr(), window(); Control statements: Decision making and branching (if..else, switch).

UNIT II : Looping (while, do .. while, for), jumping (break, continue, goto), nested loops. Functions: overview (definition, declaration), defining a function, accessing a function, function prototypes, call by value, call by reference, recursion, iteration, advantages and disadvantages of recursion over iteration, string functions (strcmp(), strlen (), strrev (), strcat (), strcpy(), strlwr (), strupr (), strstr()), Math functions (sqrt (), abs (), sin (), cos (), Standard function- exit ().

UNIT III : Arrays : Defining an array, array initialization, processing an array, passing array to a function, multidimensional arrays, arrays and strings; data files: file opening modes, character I/O(getc(), putc()), String I/O (fgets(), fputs()), Formatted console I/O(fscanf(), fprintf()).

UNIT IV : Interpolation - Existence and uniqueness theorem for polynomial interpolation, Lagrange's interpolation polynomials; difference tables – divided difference, forward difference, backward difference; Newton's forward and backward interpolation formula; differentiation – first derivative; integration – Simpson's 1/3rd rule, trapezoidal rule.

UNIT V : Newton-Raphson method; regula-falsi method, secant method, bisection method for solving polynomial equations; Gauss elimination method for solving system of equations with pivotal condensation method; numerical solution of differential equations – Euler's method, Runge-Kutta methods (up to second order) i.e, Heun's method, polygon method.

Text Books :

1. Balaguruswamy, E.: *Programming in ANSI C*, Tata McGraw Hill publication, 2002.
2. Kanetkar, Y.: *Let us C*, BPB Publication, 2016.
3. Rajaraman, V.: *Computer Oriented Numerical Methods*, PHI Learning Pvt. Ltd., New Delhi, 2002.

2. Scarborough, J.B., *Numerical Mathematical Analysis*, Oxford and IBH Publishing Ltd, 1930.
3. Rajaraman, V.: *Computer Programming in C*, PHI Private Limited, 2002.
4. Jain, M. K., Iyenger, S. R. K., Jain, R. K.: *Numerical Methods, Problems and solutions*, New Age International Publishers, 2004.
5. Kandasamy, P., Thilagavathy, K., Gunavathy, K.: *Numerical Methods*, S. Chand & Co., 2006.
6. Cheney, E.L, Kincald, D.K.: *Numerical Mathematics and computing*, Brooks/Cole Cengage Learning, 2013.

Part: (B) (Practical)
(Number of Practice/Teaching hours: 48; Marks : 40)
Part-1

The following programs are to be practiced.

1. Roots of quadratic equation $ax^2 + bx + c = 0$,
2. Arrangement a given set of numbers in increasing/decreasing order; calculation of mean,
3. Evaluation e^x, $\sin x$, $\cos x$, $\log(1 + x)$ using power series method,
4. Addition, subtraction and multiplication of matrices using function,
5. Evaluation of factorial of a positive integer and evaluation of binomial coefficients,
6. Determination of the trace of a given square matrix
7. Determination of the determinant and inverse of a given real matrix (up to order 4),
8. Checking whether a given square matrix is Symmetric (Skew-Symmetric) or not.
9. General programming :

   -Given a positive integer n, to find the sum of the digits of n. (e.g. n= 12071, sum = 11)
   -Given a positive integer n, to get the reverse of n, (e.g. n =12075,reverse = 57021)
   -Given a triangle with sides of length a, b, c, to check if the triangle is isosceles
   -Given a triangle with sides of length a, b, c, to check if the triangle is equilateral
   -To check if a point (x, y) in the plane is inside or outside a circle with centre at the origin and to find the radius of the circle.

10. Number Theory:
    -Given an integer n, to check if n is prime or not.
    -To find the gcd of two integers by Euclidean algorithm. Hence, to find the lcm of the numbers.
    -To find the gcd of n integers using Euclidean algorithm (extension of the previous program)
    -Sieve method to list prime numbers, i.e. given n to list all

1. Lagrange's interpolating Polynomial.
2. Newton forward difference interpolating polynomial.
3. Newton backward difference interpolating polynomial.
4. Simpson's 1/3 rule for numerical integration.
5. Trapezoidal rule for numerical integration.
6. Newton-Raphson method.
7. Regula-falsi method.
8. Bisection method.
9. Gauss elimination method.
10. Heun's method for solving an initial value problem.

NOTE :

(i)There will be practical examination for 40 marks of three hours duration of which 25 marks will be for Part 1 and 15 marks for Part 2. All output should be in the form of an output file.

(ii)There will be an internal exam for practical for which the full marks will be 25% of the full practical marks.

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HOP 2
Operations Research
(Number of Teaching hours: 80; Time:3 hrs; Marks: 100)
(To answer five questions, choosing one out of two questions from each unit)

UNIT I : *(i) Brief review*: Convex Sets and their properties, Linear Programming Problem (LPP), Mathematical formulation of LPP, General Linear Programming Problem, Graphical Solution Method of a LPP.
(ii) Simplex Method - Slack and Surplus variables, Standard and Canonical form of LPP, Duality in Linear Programming Problem, Simplex Method of solution of LPP: initial simplex tableau , pivot entry of a simplex tableau , calculating new tableau , terminal simplex tableau and its interpretation, algorithm of the Simplex method.

UNIT II : *Transportation problem*: General transportation problem, existence of feasible and optimal solutions, the transportation table, duality in transportation problem, loops in transportation, Linear programming formulation of the transportation problem and its solution, obtaining an initial basic feasible solution, Optimality test, degeneracy in transportation problem, exceptional cases in transportation problem.

UNIT III : *Assignment problem*: mathematical formulation of assignment problem, the assignment method, special cases in assignment problems, a typical assignment problem, the travelling salesman problem.
Markov analysis: Markov processes, state transition matrix, transition diagram, construction of a state-transition matrix, n-step transition probabilities, steady state conditions, Markov analysis algorithm.

UNIT IV : *Theory of Games*: two-person-zero–sum games, player, pure and mixed strategy, optimum strategy, value of the game, payoff matrix, the maximin–minimax principle, fair and strictly

determinable game, saddle point, rule for determining a saddle point. Game without saddle point minimax and maximin criterion for game without saddle point.

UNIT V : Graphical solution of 2xn and mx2 games, dominance property and modified dominance property, arithmetic method for nxn games, reduction of a game problem to a linear programming problem and its solution.

Text Book :

1. Swarup, K., Gupta, P. K. and Singh, M. M. : *Operations Research*, Sultan Chand & Sons, New Delhi, 2002 Edition

Reference Books:

1. Gupta, P. K., and Hira, D. S.: *Operations Research – An Introduction*, S. Chand & Co. Ltd., 2002.

2. Taha, H.A.: *Operations Research: An Introduction*, Prentice Hall of India, 2006.

3. Goel & Mittal: *Operations Research,* Pragati Prakashan, 2014.

4. Lipschutz, S.: *Theory and Problems of Finite Mathematics*, Schaums Outlines –McGraw Hill Book Company , 1983.

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HOP 3
Fluid Mechanics

(Number of Teaching hours: 80; Time:3 hrs; Marks: 100)
(To answer five questions, choosing one out of two questions from each unit)

UNIT I : Lagrangian and Eulerian methods of describing fluid motion; velocity of a fluid particle; material, local and convective derivatives; acceleration of a fluid particle; streamline motion and turbulent motion; steady flow; streamlines and path lines; velocity potential; vorticity vector; flux of a fluid; equation of a continuity by Euler's method; equation of continuity by Lagrange's method; equivalence of the Eulerian and the Lagrangian forms of the equation of continuity; equation of continuity in Cartesian, cylindrical and spherical polar coordinates; some symmetrical forms of the equation of continuity; boundary surface.

UNIT II : Euler's equations of motion (Vector and Cartesian forms); conservative field of force; pressure equation; Bernoulli's theorem (steady motion with no velocity potential and conservative field of force); Lagrange's equations of motion; equations of motion under impulsive forces (Vectors and Cartesian forms); energy equation.

UNIT III : Two dimensional motion; stream function or current function; properties of the stream function; irrotational motion ; complex potential and velocity; Cauchy-Riemann equations in polar form; images; source and sinks in two dimensions; two dimensional doublet; complex potential for a doublet; images of a simple source w.r.t. two dimensional motion; image of a source w.r.t. a line and a circle; motion symmetrical about an axis , lines of motion being in planes passing through the axis-Stoke's stream function and determination of ; image of a source w.r.t. a sphere; image of a doublet in front of a sphere; Blasius theorem.

UNIT IV : Perfect fluid; fluid pressure, pressure at a point in a fluid in equilibrium, pressure of heavy fluids; transmissibility of fluid pressure, conditions for equilibrium; pressure equation, surfaces of equal pressure; surfaces of equi-density; floating bodies-conditions of equilibrium of a floating body, freely floating bodies and bodies floating under constraint; stability of floating bodies.

UNIT V : Resultant thrust on a solid wholly or partially immersed in a heavy fluid at rest, resultant vertical thrust on a surface exposed to the pressure of a heavy fluid, resultant pressure on a surface (plane & curved)-horizontal thrust and vertical thrust, centre of pressure; gases; internal energy of a

gas, reversible isothermal change, reversible adiabatic change, atmosphere; equilibrium of an isothermal atmosphere, convective equilibrium.

Textbook:

1. Raisinghania, M.D. : *Fluid Dynamics*, S. Chand & Company Ltd., Ram Nagar, New Delhi- 110055, Sixth Edition 2005.

2. Kar, J.M. : Hydrostatic, K.P.Basu Publishing Company, 1992 Edition.

Reference Book:

1. Swarup, S. : Fluid Dynamics, Krishna Prakashan Media (P) Ltd., 2009.

2. Ray, M., Sharma, H.S.: *A Textbook of Hydrostatics*, S. Chand Publications, 1995.

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HOP 4

Complex Function Theory

(Number of Teaching hours: 80; Time:3 hrs; Marks: 100)

(To answer five questions, choosing one out of two questions from each unit)

UNIT I : Complex numbers; field properties. Absolute value, conjugate : Basic properties; Real and imaginary parts; identification of the plane with the complex numbers; distance function (in terms of the absolute value); triangle inequality, parallelogram law; related inequalities; polar representation; magnitude, argument; De Moivre's theorem; roots of unity; geometry of complex numbers : straight lines, circles in terms of complex numbers, equation of circle through three points, equation of half plane; Spherical representation.

UNIT II : Sequence of complex numbers; convergence, ratio test, root test, lim. Inf, lim. Sup, Cauchy sequence; Power series; absolute convergence, uniform convergence; Circle of convergence; Cauchy-Hadamard formula for the radius of convergence; ratio test; Analytic function; basic properties; analyticity of power series; power series definition of (complex) exponential function, sine, cosine function; branch of logarithm; principal branch of a power function $z^{(\alpha)}$ for complex number α.

UNIT III : Differentiability, Cauchy Riemann theorem and its converse; integration of complex valued function along a piece-wise differentiable curve (using real integral for real and imaginary parts); basic properties (including inequalities); Cauchy's theorem and its corollaries.

UNIT IV: Cauchy's Integral formula for a disc, power series representation of analytic functions, Cauchy's estimate; Liouville's theorem; fundamental theorem of Algebra; zeroes of analytic functions; related results; Maximum modulus theorem.

UNIT V: Analytical mapping as function, stereographic projection; conformal maps; Mobius transformations, cross ratio; images of regions under elementary analytic function.

Text Books:

1. Conway, J.B.: *Functions of one complex variable*, Springer New York, 2012.
2. Ahlfors, L.V.,: *Complex Analysis* (2nd Edition) –, McGraw-Hill Education, 1990.
3. Ponnusamy, S., and Silverman, H.,: *Complex Variables and Applications*, Birhkäuser, 2006.

Reference books:

1. Churchill, R. V.: *Complex Variables and Applications*, McGraw-Hill Education, 1996.
2. Copson, E.T.: *An Introduction to the Theory of functions of a complex Variable*, Oxford University Press, 1995.

3. Shastri, A.R.: *An Introduction To Complex Analysis*, Macmillan India Ltd., 2003.

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HOP 5
Discrete Mathematics

(Number of Teaching hours: 80; Time:3 hrs; Marks: 100)
(To answer five questions, choosing one out of two questions from each unit)

UNIT I : Mathematical induction, principle of inclusion and exclusion, pigeon hole principle, finite combinatorics, generating functions, partitions, recurrence relations and recursive algorithms, linear recurrence relations with constant coefficients, homogeneous solutions, total solution, solution by the method of generating functions.

UNIT II : Definition, examples and basic properties of ordered sets, duality principle, lattices as ordered sets, lattices as algebraic structures, sub lattices, products and homomorphisms.

UNIT III : Definition, examples and properties of modular and distributive lattices, Boolean algebras, Boolean polynomials, minimal-forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, switching – circuits and applications of switching circuits.

UNIT IV : Definition, examples of graphs, degree of a vertex; handshaking lemma; paths, circuits, cycles, complete graphs, bipartite graphs, isomorphism of graphs, connected graphs, Eulerian paths, Euler's theorem, Hamiltonian paths, shortest path, travelling salesman problem.

UNIT V : Adjacency and incidence matrices, block, cut points, bridges, connectivity, trees and characterization of trees, Euler's formula for connected planar graphs.

Text Books:

1. Liu C.L. : *Elements of Discrete Mathematics* (2nd Edition), McGraw Hill Education (India) Pvt Limited, 2000.
2. Sen M.K. and Chakraborty B.C. : *Introduction to Discrete Mathematics*, Books and Allied Ltd, 2012.
3. Davey B.A. and Priestley H.A. : *Introduction to Lattices and Order*, Cambridge University Press, Cambridge, 2002.
4. Goodaire E.G. and Parmenter M.M. : *Discrete Mathematics with Graph Theory* (2nd edition), Pearson Education Pvt Ltd, Indian Reprint, 2003.
5. Sankar S.K., : *A Text book of Discrete Mathematics*, S.Chand, 2008.
6. Mittal H. and Goel V.Kr. : *A Text Book of Discrete Mathematics*, I.K. International, 2010.

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HOP 6
Probability Theory

(Number of Teaching hours: 80; Time:3 hrs; Marks: 100)
(To answer five questions, choosing one out of two questions from each unit)

UNIT I : Random experiment, sample space, simple and composite events ; classical definition of probability and the theorems of total and compound probability based on this definition, Axiomatic approach to the notion of probability, important theorem based on this approach; conditional probability and compound probability; independent events; Bayes' Theorem: statement, proof and related problems.

distribution, related problems.

UNIT III : Mathematical expectation of a random variable, theorems on the sum and product of several random variables. Moments as expectations, mean, variance and standard deviation, conditional expectation and conditional variance, covariance; moment generating function, characteristic functions, properties and other theorems of characteristic functions; Chebyshev's inequality, weak law of large numbers; problems involving Chebyshev's inequality.

UNIT IV : Distributions: Distribution functions:- geometric, hypergeometric, binomial and Poisson distributions and their elementary properties; Moments of these distributions upto 4th order, Recurrence relation for the moments of Binomial distribution, Poisson distribution. Normal distribution, its properties, mean, median, mode, related problems. Poisson distribution and normal distribution as the limiting case of binomial distribution.

UNIT V : Chi-square, F and t –distributions, their derivation and important properties; Stochastic convergence (convergence in probability); statement of Central Limit Theorem in case of equal components, sampling distribution of the mean and variance of independent random observations from a normal population.

Text Books:

1. Kapur, J.N., Saxena, H.C,: *Mathematical Statistics*, S. Chand and Co., 2010.
2. Gupta, S.C. and Kapoor, V.K., : *Fundamentals of Mathematical Statistics*, S. Chand and Co, 2007.

Reference Books:

1. Gun, A.M., Gupta, M.K., Dasgupta, B.: *Fundamentals of Statistic-Vol I*, World Press Pvt Ltd, 2013.
2. Medhi, J.: *Statistical Methods – An introductory Text*, New Age International Publishers, 2006.
3. Spiegel, M.R. and Stephens, L.J.: *Probability and Statistics-Schaum's outline series*, McGraw Hill Education, 2017.

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HOP 7
Elementary Differential Geometry

(Number of Teaching hours: 80; Time:3 hrs; Marks: 100)
(To answer five questions, choosing one out of two questions from each unit)

UNIT I : Level curves, parametrised curves, tangent vectors, arc-length, reparametrisation, level curves vs. Parametrised curves.

UNIT II : Curvature, plane curves: signed unit normal, signed curvature, centre of curvature, osculating circles, space curves: principal normals, binormals, torsions, Serret-Frenet equations.

UNIT III : Simple closed curves, interior and exterior of a simple closed curve, statement of Jordan Curve theorem, the isoperimetric inequality, the four vertex theorem.

UNIT IV : Surface, smooth surface, tangents, normals, orientability, examples of surfaces: generalised cylinder, generalised cone, ruled surface, surface of revolution, quadric surfaces.

UNIT V : Lengths of curves on surfaces, the first fundamental form, Isometries of surfaces, conformal mappings of surfaces, surface area, equiareal maps and a theorem of Archimedes.

Text Book:

1. Pressley, A.N.: *Elementary Differential Geometry* (Sixth Indian reprint), Springer International Edition, 2014.

Reference Books:

1. Manfredo P. do Carmo: *Differential Geometry*, Curves and Surfaces, Dover Publications, New York, 2016.
2. O'Neill, B.: *Elementary Differential Geometry* (Revised Second Edition), Elsevier, 2006.

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