

DRAFT SYLLABUS FOR B.A./B. Sc. (MATHEMATICS), September,2012

Course Structure: Semester-wise distribution.

First Semester (Total marks: General –100)

GHS 11 : Algebra-I, & Calculus – I, (100 marks, 80 lectures)

Second Semester (Total marks: General – 100)

GHS 21 : Geometry & Vector Calculus (100 marks, 80 lectures)

Third Semester (Total marks: General – 100)

GHS 31: Algebra II, & Calculus – II (100 marks, 80 lectures)

Fourth Semester (Total marks: General – 100) (100 marks, 80 lectures)

GHS 41: Statics & Dynamics. (100 marks, 80 lectures)

Fifth Semester (Total marks: Honours - 200)

H 51 : Elementary Number Theory & Advanced Algebra (100 marks, 80 lectures)

H 52: Differential Equations & Advanced Dynamics . (100 marks, 80 lectures)

Sixth Semester (Total marks: Honours - 200)

H 61 : Advanced Calculus (100 marks, 80 lectures)

HOPT62: Optional paper (100 marks, 80 lectures)

Optional Papers : Any one of the followings (100 marks, 80 lectures)

OP 1 : Computer programming in C & Computer Oriented Numerical Analysis

OP 2 : Operations Research

OP 3 : Hydro Mechanics

OP4 : Financial Mathematics

OP5 : Discrete Mathematics

OP6 : Mathematical Modeling

(Abbreviation: G = general, H = honours, GH = general and honours)

GHS 11

ALGEBRA-I & CALCULUS – I

(Number of Teaching hours: 80; Time:3 hrs; Marks: 100)

(To answer five questions, choosing one out of two questions from each unit)

UNIT I : Brief review of basics in set theory such as ways of describing a set, set operations, empty set, disjoint sets, De Morgan's laws, Venn diagrams; power sets, cartesian products, cardinality results; relation as a subset of cartesian product (notation: xRy if $(x,y) \in R$); relation on a set: reflexive, symmetric, anti-symmetric, transitive; examples from geometry and number systems; equivalence relation and equivalence classes; partitions.

Functions and graphs: real valued functions such as polynomials, rational functions, logarithmic functions, exponential functions, hyperbolic functions; limits, $\epsilon - \delta$ definition, standard theorems on limits, standard limits; continuity : Intuitive idea, $\epsilon - \delta$ definition, theorems on sum, difference, product, quotient and composite of continuous functions; discussion of continuity of the functions mentioned earlier and their composites.

UNIT II : Brief review of functions/mappings, inclusion map; restriction of a map; composition of maps; associativity; onto, one-one, bijective maps; inverse images of sets, inverse of a bijective map; finite and infinite sets; Proof of " if A is a finite set then $f : A \rightarrow A$ is one-one if and only if f is onto"; examples where this assertion does not hold. A brief review of $m \times n$ matrix over R/C as a rectangular array of numbers (motivation through systems of linear equations); transpose, conjugate transpose; definition of inverse of a matrix; special type of matrices: diagonal, scalar, upper/lower triangular, nilpotent, idempotent, symmetric, skew symmetric, hermitian, skew hermitian matrices; trace of a square matrix; row vectors and column vectors of a matrix; row rank/column rank of an $m \times n$ matrix (in terms of linear independence of row/column vectors of the matrix); Adjoint of a matrix; inverse in terms of adjoints; determinantal rank of matrix; equality of rank and determinantal rank; Elementary operations; elementary matrices; row/column reduced echelon form of a matrix; determination of the inverse of a matrix by elementary operations; theorem on the equality of row-rank and column-rank; rank of a matrix; determination of the rank by elementary operations;); systems of linear equations: homogeneous and non-homogeneous;

UNIT III : Properties of continuous functions defined on closed and bounded intervals : (statements with illustrations only for the following) boundedness, intermediate value theorem, uniform continuity . Derivatives of real valued functions on intervals : definition; Derivative as a rate measurer, derivative as the gradient of tangent (geometrical interpretation only); Theorems on sum, difference, product, quotient and composite of differentiable functions.

Review of methods of differentiation; successive differentiation; Liebnitz's theorem; L'Hospital's Rule (statements only with applications).

UNIT IV. Anti-derivative : review of the standard methods; integration by parts and by partial fractions; integral of a continuous function as the limit of Riemann sum (including sums arising out of unequal distribution of interval); examples of evaluation of integrals from the definition. Definite Integrals, fundamental theorem of integral calculus and differentiability of integrals of continuous functions (statements with illustrations only) properties of definite integral, evaluation of integrals using these properties; reduction formulas for $\int \sin^n x dx$, $\int \cos^n x dx$, $\int \tan^n x dx$, $\int e^{ax} \sin(mx) dx$, $\int e^{ax} x^n dx$, $\int e^{ax} (\log x)^n dx$, $\int \sin^n x \cos^m x dx$ and their combinations; improper integrals, convergence and evaluation from definition.

UNIT V. Formation of differential equations; equations of first order and first degree; solutions by separation of variables, by substitution; homogeneous equations; linear equations; Bernoulli's equation; exact equations; reduction to exact form by integrating factors; differential equations of first order but higher degrees; Clairut's equation and singular solution; geometrical interpretation applications of first order differential equations to geometric and physical problems (simple cases only) including orthogonal trajectories'

Books

Text Books:

1. Bhattacharya, P. B., Jain, S. K., and Nagpaul, S. R. : Basic Abstract Algebra, Cambridge Press, 1995 Edition.
2. Saikia, P. K.: Linear Algebra, Pearson, Delhi, 2009 Edition.
3. Maity, K. C. and Ghosh, R. K. : Differential Calculus, New Cental Book Agency Pvt Ltd., 2001 edition.
4. Das, B.C. and Mukharjee, B.N., Calculus , UN Dhar and Sons Publisher.
5. Thomas, G. B., and Finney, R. L. : Calculus and Analytic Geometry, Narosa Publishing House, 2002 Edition.

Reference Books

1. Fraleigh, J. B.: A First Course in Abstract Algebra, Narosa Publishing House, 1999 Edition.
2. Stewart, J.: Essential Calculus Early Transcendentals, Thomson Brooks/Cole, USA, 2007 Edition.

GHS 21

Geometry & Vector Calculus

(Number of Teaching hours:80; Time: 3hrs; Marks:100)

(To answer five questions, choosing one out of two questions from each unit)

UNIT I : Change of axes – invariants; pairs of straight lines; general equation of second degree; the standard form; reduction of the general equation to standard form; conditions for different conics; General conics : equations of tangents, normals, pairs of tangents, chord of contact, chord in terms of middle points, pole, polar, conjugate lines, diameter , asymptotes.

UNIT II: Polar equation, equation of a conic, directrix, chord, tangent and normal; parabola, ellipse, hyperbola; conjugate diameters of ellipse and hyperbola; rectangular hyperbola; conjugate hyperbola. Space co-ordinates: rectangular, cartesian, cylindrical, spherical, and polar; equation of planes; angle between two planes; perpendicular distance of a point from a plane;

UNIT III : Equations of straight lines in space; co-planarity of two straight lines; perpendicular distance of a point from a straight line; shortest distance between two straight lines in space; Sphere – plane section and its equation; sphere through a given circle; tangent plane; pole and polar plane; intersection of two spheres; Equation of a cone with a conic as a guiding curve; enveloping cone; mutually perpendicular generators; tangent planes; reciprocal cone; right circular cone; equation of a cylinder with a conic as a guiding curve; right circular cylinder.

UNIT IV : Products (scalar and vector products) of two, three and four vectors – properties, geometrical significance and applications; Vector valued functions (up to 3 variables); derivatives of such a function of a single variable; properties and geometrical applications; arc length, unit tangent vector; curvature, normal vector; derivatives of scalar and vector products;

UNIT V: Velocity and acceleration in cartesian co-ordinates, radial and transverse accelerations on smooth curves (simple problems only), tangential and normal components of velocity and acceleration. Directional derivatives, gradient of a scalar- valued function, tangent planes; vector fields, curl and divergence of a vector field, Physical and geometrical interpretation and elementary properties.

Books

Text Books:

1. Das, B.: Analytical Geometry and vector Analysis, Orient Book Co., Calcutta, 1998 Edition.
2. Narayan S.: Analytical Solid Geometry, S. Chand & Co., New Delhi, 2003 Edition.
3. Thomas, G. B., and Finney, R. L.: Calculus and Analytic Geometry, Narosa Publishing House, 2002 edition.
4. Narayan, S. and Mittal, P. K.: A Text Book of Vector Analysis, S. Chand & Co. Ltd., New Delhi, 2003 Edition.
6. Chorlton, F.: Text Book of Dynamics, CBS Publishers and distributors, Delhi, 2002 Edition.
7. Chatterjee, D.: Vector Analysis, PHI Learning Pvt Ltd. New Delhi, 2002 Edition.
8. Ghosh, R. K. and Maity, K. C.: Vector Analysis, New Central Book Agency, 2001 Edition.

Reference Book

1. Stewart, J.: Essential Calculus Early Transcendentals, Thomson Brooks/Cole, USA, 2007 Edition.

GHS 31

Algebra II & Calculus – II

(Number of Teaching hours: 80; Time:3 hrs; Marks: 100)

(To answer five questions, choosing one out of two questions from each unit)

UNIT I : Binary operations as maps from $A \times A \rightarrow A$; commutative and associative binary operations; identities and inverses (one-sided as well as two sided); examples; groups: definition; examples of groups such as $Z, Q, R, C, Q^*, R^*, C^*, Z_n, S_n, M_2(R), GL_2(R), R^2, R^3, n^{\text{th}}$ roots of unity etc; laws of indices in both additive and multiplicative notation; right and left cancellation laws; uniqueness of identity and inverses; group tables of groups of low order (up to 8); subgroups :examples; cyclic subgroup generated by an element; cyclic group; abelian group; subgroups of cyclic groups
Determination of all subgroups of Z ;
Order of an element; examples of elements of finite order; of infinite order; coset with respect to a subgroup in a group; cosets as equivalence classes; Z_n as cosets; Lagrange's theorem and its applications; groups of prime order; Fermat's (little) theorem; Euler's generalizations; application to simple number theoretic problems.

UNIT II: Polynomials over $Z / Q / R / C$; addition and multiplication; degree of a polynomial; degree of sum and product of polynomials; the division algorithm; remainder theorem; gcd; the Euclidean algorithm; irreducible polynomials; application of Eisenstein's theorem; unique factorisation theorem; roots of a polynomial; factor theorem; fundamental theorem of algebra (statement only); its failure for polynomials over $Z / Q / R$; simple tests of irreducibility of polynomials with rational / integer coefficients; detailed study of roots of a polynomial with real coefficients: immediate consequence of the fundamental theorem of algebra; multiple roots, common roots, complex roots, surd roots;
Relation between roots and coefficients of a polynomial; symmetric function of the roots of a polynomial equation with special reference to cubic and biquadratic equations; transformation of equation; Descartes' rule of signs – simple applications; location of roots using Rolle's theorem; solution of a cubic by Cardan's method; De Moivre's theorem – application to solution of equation.

UNIT III: Sequences of real numbers : definitions of bounded sequence, convergent sequence, limit of a sequence, monotonic sequence; examples; proof of the fact that monotonic and bounded sequences are convergent (using completeness of R as an axiom); Cauchy sequence; Cauchy's general principle of convergence;
Infinite series of real numbers: partial sums, convergent series, comparison test, ratio test, Raabe's test, root test; absolute convergence; Leibnitz's theorem for alternating series; power series; radius of convergence (without the notion of limit superior), standard examples of power series.

UNIT IV : Application of differential calculus: Sign of the derivatives of a real valued function of a real variable, vanishing of $f'(x)$; Rolle's theorem; geometric interpretation, mean value theorems; applications of the mean value theorems: (i) increasing and decreasing functions, (ii) concavity upwards and downwards, (iii) points of inflections, multiple roots. use of differentials in approximation and error estimates; maxima and minima; asymptotes; curvature of plane curves (cartesian and parametric equations only);
Real-valued functions of two or three variables: limits; continuity; partial derivatives of first and second orders; Schwarz's theorem (statement only); differentials; chain rules; Euler's theorem on homogeneous functions, proof upto three variable case.

UNIT V: Location of roots of $f'(x)$, proof of the fundamental theorem of integral calculus; Taylor's and Maclaurin's theorem with Cauchy's form of remainders; Taylor's and Maclaurin's series; expansion of standard functions such as e^x , $\sin x$, $\cos x$, $\log(I+x)$, $(I+x)^n$.

Applications of integral calculus : determination of (i) areas under simple plane curves, (ii) lengths of simple plane curves, (iii) volume and surface areas of solids of revolution in standard cases. Evaluation of line integrals (in a plane); double integrals; change of order of integration; application in determination of area, volume (simple cases only).

BOOKS

Text Books :

1. Bhattacharya, P. B., Jain, S. K., and Nagpaul, S. R. : Basic Abstract Algebra, Cambridge Press, 1995 Edition.
2. Gopalakrishnan N.S., University Algebra, New Age International Publishers, 1986 Edition.
3. Maity, K. C. and Ghosh, R. K.: Differential Calculus, New Cental Book Agency Pvt Ltd., 2002 Edition.
4. Das, B.C. and Mukharjee B.N., Differential Calculus , UN Dhar and Sons Publisher.
5. Thomas, G. B., and Finney, R. L.: Calculus and Analytic Geometry, Narosa Publishing House, 2002 Edition.
6. Maity, K. C. and Ghosh, R. K.: Integral Calculus, New Cental Book Agency Pvt. Ltd., 2002 Edition.
7. Das, B. : Higher Algebra, Asoke Prakashan, Calcutta, 2000 Edition.
8. Narayan S.: A course of Mathematical Analysis, S.Chand and Co., 1962 Edition.

Reference Books

1. Fraleigh, John B. : A First Course in Abstract Algebra, Narosa Publishing House, 1999 Edition.
2. Stewart, J.: Essential Calculus Early Transcendentals, Thomson Brooks/Cole, USA, 2007 Edition.
3. Bernside, W. S., and Panton, A. W.: Theory of Equations, Vol. I, S. Chand & Co., New Delhi, 2000 Edition.

GH S 41

Statics & Dynamics

(Number of Teaching hours: 80; Time:3 hrs; Marks: 100)

(To answer five questions, choosing one out of two questions from each unit)

UNIT I: Composition and resolution of forces; parallelogram of forces, Components and resolved parts, Coplanar forces: Equilibrium of concurrent forces, Triangle of forces, Lami's Theorem and its converse. Parallel forces. Moment of a force ; Definition, geometrical representation of Moments, Varignon's Theorem. Couples ;definition, equilibrium of Couples, Equivalence of two Couples, Resultant of Couples, Resultant of a couple and a force.

UNIT II: Reduction of coplanar forces, equilibrium of coplanar forces. Friction: laws of statical friction, laws of limiting friction, solution of problems on equilibrium of heavy bodies (such as uniform rods) resting on plane surfaces.
Centre of gravity: c.g of thin uniform rod, uniform lamina, triangular lamina and lamina in the form of a parallelogram and trapezium.

UNIT III: Rectilinear motion with variable Laws of forces; Force of repulsion varying as displacement, Motion under inverse square Law, Motion of a particle attracted towards the centre of the Earth. Motion under other laws of forces, simple harmonic motion; velocity and acceleration , Amplitude, Time-period. Collision of elastic bodies; direct and oblique impact, Loss of Energy due to collision, impulsive action between colliding spheres .

UNIT IV: Projectiles; Horizontal Range , Time of flight, Greatest height , position and velocity at any time, path of a projectile is a parabola.
Rectilinear motion in resisting media on a horizontal plane where resistance varies as (i) velocity, (ii) square of velocity, (iii) displacement; vertical motion under gravity where resistance varies as (i) velocity, (ii) square of velocity.

UNIT V: Tangential and normal acceleration on smooth curves, radial & transversal acceleration, motion on a smooth plane curve such as vertical circles and cycloids.
Impulse and Impulsive force, conservation of linear momentum. Work done by a force; work energy equation; potential function; conservative forces.

BOOKS

Text Books:

1. Gupta, P. K., and Juneja, R. : Dynamics, Ramesh Book Depot, Jaipur, 2003 Edition.
2. Ray, M. : A Text Book on Dynamics for B.A./B.Sc. students, S. Chand Publication, Delhi, 2002 Edition.
3. Singh, K. K. : Text Book of Dynamics, PHI Learning pvt. Ltd., New Delhi, 2011 Edition.
4. Das B.C. and Mukharjee B.N., Dynamics UN Dhar and Sons Publisher, 2002 Edition.
5. Das, B. C. and Mukherjee, B. N.: Statics, U. N. Dhar & Sons Publications, Kolkata, 2002 Edition.
6. Loney, S. L.: An elementary treatise on the Dynamics of a particle and of rigid bodies, Rahda publishing House, Kolkata, 2000 Edition.

Reference Books

1. Varma, R. S.: Statics, Pothishala, Allahabad, 2001 Edition.
2. Loney, S. L.: An elementary treatise on Statics, Rahda publishing House, Kolkata, 2000 Edition.
3. Singh, K. K. : Text Book of Dynamics, PHI Learning pvt. Ltd., New Delhi, 2011.

Elementary Number Theory & Advanced Algebra

(Number of Teaching hours: 80; Time:3 hrs; Marks: 100)

(To answer five questions, choosing one out of two questions from each unit)

UNIT I: Divisibility in the set of integers; basic properties; the division algorithm; gcd; elementary properties; the Euclidean algorithm; lcm; primes (in the set of natural numbers); fundamental theorem of arithmetic; Euclid's proof of the infinitude of primes; arbitrary gaps in the distribution of primes; Congruences in the set of integers modulo a positive integer; basic properties; complete residue system; reduced residue system; Euler's ϕ - function; Fermat's theorem; Euler's generalization of Fermat's theorem; applications, Wilson's theorem.

UNIT II: Solution of congruences; linear congruences; Chinese remainder theorem; congruences of higher degree modulo a prime.

Some functions of Number Theory-- Greatest integer function; elementary properties; Arithmetic functions; multiplicative functions; functions such as $\phi(n)$, $\mu(n)$, $\tau(n)$, $\sigma(n)$, $\sigma_k(n)$;

UNIT III: Normal subgroups, examples; conditions for a subgroup to be normal; center of a group; examples; quotient group; homomorphism, kernel and image of homomorphism, isomorphism of groups – examples and elementary properties. Fundamental theorem of group homomorphism; isomorphism theorems; automorphisms; inner automorphisms; examples; rings (motivation through \mathbf{Z}): definitions and examples of (i) rings with identity, (ii) commutative rings, (iii) rings with and without zero-divisors, integral domains, (iv) division rings (v) fields (examples to include \mathbf{Z}_p , integers mod p , fields \mathbf{Q} , \mathbf{R} , \mathbf{C} , polynomial rings $R[x]$, matrix rings $M_n(\mathbf{R})$); basic properties of rings; characteristic of rings; finite integral domains; (\mathbf{Z}_p as an example);

Subrings; ideals: right, left and two-sided; generated by a subset, more specifically by a finite number of elements in a commutative ring with 1; principal ideals; examples of ideals in \mathbf{Z} , \mathbf{Z}_n , $M_n(\mathbf{R})$; prime ideals, maximal ideals in a commutative ring with 1; examples; quotient ring, \mathbf{Z}_n as a quotient ring.

UNIT IV: Principal ideals; examples of ideals in \mathbf{Z} , \mathbf{Z}_n , $M_n(\mathbf{R})$; prime ideals, maximal ideals in a commutative ring with 1; examples; quotient ring, \mathbf{Z}_n as a quotient ring. Ring homomorphisms; kernels; isomorphism; homomorphisms and isomorphism theorems including the correspondence theorem; determination of ideals in \mathbf{Z}_p ; divisibility in integral domains (with 1); units, associates, prime elements, irreducible elements, gcd, Euclidean domain, principal ideal domain, unique factorisation domains – definition, examples and basic results

UNIT V: Vector spaces (motivation through \mathbf{R}^2 , \mathbf{R}^3) – examples, basic properties; subspaces; homomorphisms or linear maps between vector spaces; isomorphisms; standard homomorphism and isomorphism theorems; direct sum (internal and external); linear dependence and independence; basis, dimension; vector space axioms for the set $L(V, W)$ of linear maps from V to W ; rank and nullity of a linear transformation; "rank + nullity = dimension" theorem. Matrix- representation of linear transformations; similar matrices, change of basis theorem (without proof, statement only and its application); ; equality of rank of a linear transformation and rank of the associated matrix.

Books

Text Books:

1. Niven, I., Zuckerman, H.S., and Montgomery, H. L. : An introduction to the Theory of Numbers, Wiley Eastern Ltd., 2000 Edition.
2. Burton, David M. : Elementary Number Theory, Universal Book Stall, 2001 Edition.
3. Herstein, I. N.: Topics in Algebra, Vikas Pub. House, 1988 Edition (reprint 1998).
4. Fraleigh, J. B. : A First Course in Abstract Algebra, Narosa Publishing House, 1999 Edition.
5. Saikia, P. K.: Linear Algebra, Pearson, Delhi, 2009 Edition.

Reference Book:

1. Bhattacharya, P.B., Jain, S. K., and Nagpaul, S. R.: A First Course in Linear Algebra, Wiley Eastern Publication, 2001 Edition.
3. Hoffman, K. and Kunze, R.: Linear Algebra, second edition, PHI Learning Pvt Ltd, New Delhi, 1971 Edition (reprint 1996).
4. Telang, S. G. : Number Theory, Tata McGraw-Hill, New Delhi, 1996 Edition.

H 52

Differential Equations & Advanced Dynamics .

(100 marks, 80 lectures)

(To answer five questions, choosing one out of two questions from each unit)

UNIT I: Linear equations of second and third order with constant coefficients – complementary functions and particular integrals for $x^n e^{ax}$, $e^{ax} \sin(mx)$, $e^{ax} \cos(mx)$, $x^n \sin(mx)$, $x^n \cos(mx)$; equations of type $a_1 x^2 y'' + a_2 x y' + a_3 y = f(x)$;

Linear differential equations of second order with variable coefficients; homogeneous equations; exact equations; transformation of the equation by changing the dependent variable/the independent variable, Normal form

Method of variation of parameters; simultaneous equations; total differential equation $Pdx + Qdy + Rdz = 0$

UNIT II: Partial differential equation. Formation of equation, solutions of linear equations of first order, Lagrange's methods, Non linear partial differential equations of first order- Standard forms I, II, III & IV.

Integral surfaces passing through a given curve, orthogonal surfaces, non-linear equations of first order, Charpit's method.

(Introduction to the following concepts should be made in vector as well as Cartesian method)

UNIT III: Motion on a rough curve; the cycloid and its dynamical properties: cycloidal motion with resistance; Central forces, Central orbit; Centre of force; motion of a particle under a central force; description of a central conic under a central force; use of reciprocal polar coordinates; stability of a nearly circular orbit.

Use of pedal coordinates and pedal equations; apse; apsidal distance; apsidal angle; perihelion and aphelion; Kepler's laws of planetary motion and its deductions; a more accurate form of the third law.

UNIT IV: Moments and products of inertia; uniform rod, a rectangular lamina, a parallelepiped, a circular ring and disc; theorems of parallel and perpendicular axes about a fixed axis; principal axes (only definition), vector angular velocity of a rigid body; vector angular momentum of a rigid body about a fixed point, principal axes; kinetic energy of a rigid body rotating about a fixed point; momental ellipsoid; equipomental systems; coplanar distributions; general motion of a rigid body.

UNIT V: Motion of a rigid body in two-dimensions; Problems illustrating the laws of motion, motion of a uniform solid circular cylinder down a rough inclined plane; motion of a circular hoop on a rough inclined plane; laws of conservation of angular momentum; problems illustrating the laws of conservation of angular momentum.

The law of conservation of energy; problems illustrating the law of conservation of energy; Impulse of a force; problems illustrating impulsive.

Books

Text Books:

1. Raisinghania, M.D.: Ordinary and Partial Differential Equations, S. Chand & Co. Ltd., New Delhi, 2002 Edition.
2. Piaggio, I.: An Elementary Treatise on Differential Equations and Applications, G. Bell & Sons, 2000 Edition.
3. Sneddon, I. N.: Elements of Partial Differential Equation, McGraw Hill. International Edition 1957.
4. Chorlton, F.: Text Book of Dynamics, CBS Publishers and distributors, Delhi, 2002 Edition.
5. Loney, S. L.: An Elementary Treatise on The Dynamics of a Particle and of Rigid Bodies, Rahda publishing House, Kolkata, 2002 Edition (only for Unit I).

Reference Book

1. Coddington, Earl A.: An Introduction to Ordinary Differential Equations, PHI Learning Pvt. Ltd., New Delhi, 1998 Edition.
2. Ramsay, A. S.: Dynamics, Part I, Cambridge University Press, 1993 Edition.
3. Singh, K. K. : Text Book of Dynamics, PHI Learning pvt. Ltd., New Delhi, 2011.

H 61

Advanced Calculus

(100 marks, 80 lectures)

(To answer five questions, choosing one out of two questions from each unit)

UNIT I: Riemann integral of functions of one variable; Darboux's theorem (statement and application); conditions for integrability; classes of bounded and integrable functions; properties of integrable functions; inequalities for integrals; functions defined by integrals; their continuity and differentiability; Mean value theorems for integrals.

Improper integrals; test for convergence when the integrand is non-negative; absolute convergence; tests for absolute and conditional convergence, beta and gamma functions; Abel's theorem, Dirichlet's theorem; Frullani's integral.

UNIT II: Integrals as functions of parameters; continuity, differentiability and integrability of such a function; applications to evaluation of integrals

Improper integrals as functions of a parameter; uniform convergence and tests for uniform convergence; continuity, differentiability and integrability of uniformly convergent improper integrals of continuous functions involving parameters; evaluation of integrals;

UNIT III: Line integral in \mathbb{R}^2 ; Riemann integral of real valued functions of two variables; evaluation of double integrals – change of order of integration; change of variable and simple problems; Green's theorem in \mathbb{R}^2 , Surface Integral and Stokes Theorem, Volume integral and Gauss's divergence theorem (statements and applications only)

UNIT IV: Basic properties of Euclidean distance function in \mathbb{R}^n ; neighbourhoods, open sets, closed sets, limit points, interior points in \mathbb{R}^n ($n = 1, 2, 3$); Bolzano-Weierstrass theorem; Cantor intersection theorem (nested interval) ; Lindelof covering theorem.

Compact sets; Heine-Borel theorem; equivalent statements; study of maps from subsets of $\mathbb{R}^n \rightarrow \mathbb{R}^m$ ($m, n = 1, 2, 3$): continuity, in terms of ϵ - δ notation; in terms of inverse images of open and closed sets; elementary properties of continuous functions; continuous functions on compact sets; special cases of continuous real valued functions on closed, bounded intervals of \mathbb{R} : bounds.

UNIT V: Intermediate value theorem; uniform continuity; discontinuities of real valued functions; monotonic functions; continuity of the inverse of a strictly monotonic function.

\mathbb{R}^m -valued functions of two or three variables ($m = 1, 2, 3$); partial derivatives; directional derivatives; total derivative, Jacobian; change in the order of partial derivatives, statements of Young's Theorem, Schwarz's Theorem and their applications, differentiation of composite functions; chain rule.

BOOKS

Text Books:

1. Narayan, S.: A Course of Mathematical Analysis, S. Chand. Delhi, 2003 Edition.
2. Apostol, Tom A. : Mathematical Analysis, Narosa Publishing House, 2002 Edition.

Reference Books

1. Stewart, J.: Essential Calculus Early Transcendentals, Thomson Brooks/Cole, USA, 2007 Edition.

2. Bartle, R. G., and Sherbert, D. R. : Introduction to Real Analysis, John Wiley & Sons, Inc, 2000 Edition.
3. Rudin, W. : Principles of Mathematical Analysis, Mc Graw–Hill Publications, 1976 International Editions (Reprint 1996).
4. Malik, S. C. and Arora, S. : Mathematical Analysis, New Age International (P) Ltd., 1992 Edition (Reprint 2001).
5. Ghosh, R.K. and Maity, K.C.: Introduction to Analysis, New Central Book Agency (P) Ltd, 2002 Edition.

HOPT 62: OP 1

Computer programming in C & Computer Oriented Numerical Analysis
(Theory)

Note: This paper consists of 2 parts A, B. Part A is theory paper consisting of 60 Marks & Part B is Practical Paper consisting of 40 Marks.

Part : A (Theory)

(Number of Teaching hours: 48; Time : 3 hrs; Marks: 60)
(To answer five questions, choosing one out of two questions from each unit)

UNIT I. C fundamentals: The C character set, identifiers and keywords, Data types, constants, variables and arrays, declarations, symbolic constants, Operators (Arithmetic, unary, relational, logical, bitwise, assignment), expressions, statements, C program structure, Need of header files, Process of compiling and running a C program; I/O functions: Header files (stdio.h, conio.h) getch(), getche(), getchar(), putchar(), scanf(), printf(), gets(), puts(), clrscr(), window(); Control statements: Decision making and branching (if..else, switch), Decision making and looping (while, do .. while, for), Jumping (break, continue, goto), Nested loops.

UNIT II. Functions: Overview (definition, declaration), defining a function, accessing a function, function prototypes, call by value, call by reference, recursion, iteration, Advantages and disadvantages of recursion over iteration, Storage classes (Automatic, Register, External, Static), String functions (strcmp (), strlen (), strrev (), strcat (), toupper (), tolower ()), Math functions (sqrt (), abs (), sin (), cos ()), Standard function- exit (), Memory allocation functions (malloc (), free (), realloc(), calloc()).

UNIT III Arrays and Pointers: Defining an array, array initialization, processing an array, passing array to a function, multidimensional arrays, arrays and strings, pointer declarations, passing pointer to a function, pointer and one dimensional arrays, Operation on pointers, functions returning pointers; Data files: File opening modes, character I/O(getc(), putc()), String I/O (fgets(), fputs()), Formatted console I/O(fscanf(), fprintf()), text mode versus binary mode, Unformatted console I/O functions – record I/O(fread(), fwrite(), ftell(), fseek(), rewind(), rename()), Record operations (append, delete, update, search, display, sorting of records).

UNIT IV: Floating point representation of numbers, Arithmetic operations with normalised floating point numbers, Errors of numbers, Binary representation of numbers; Interpolation – Lagrange's interpolation polynomials; difference tables – divided difference, forward difference, backward difference; Newton's forward and backward interpolation formula; Differentiation – first derivative; integration – simpson's 1/3rd rule, trapezoidal rule.;

UNIT V: Newton-Raphson method; regula-falsi method, secant method, bisection method for solving polynomial equations; Gauss elimination method for solving system of equations; numerical solution of differential equations – Euler's method, Runge-Kutta methods (up to second order) i.e; Heun's method, polygon method

Books:

Text Books :

1. Kanetkar, Y.: Let us C, B. P. B Publication, 1993 Edition.
2. Gottfried, B. S.: Theory and Problems of Programming with C, Tata McGraw Hill Publication, 1998 Edition.
3. Rajaraman, V.: Computer Oriented Numerical Methods, PHI Learning Pvt. Ltd., New Delhi, 2002 Edition.
4. Scarborough, J.B., Numerical Mathematical Analysis, Oxford and IBH Publishing Ltd. New Delhi, 1930 Edition.

Reference Books:

1. Balaguruswamy, E.: Programming in ANSI C, Tata McGraw Hill publication, 2002 Edition.
2. Rajaraman, V.: Computer Programming in C, PHI Private Limited, New Delhi, 2002 Edition.
3. Jain, M. K., Iyenger, S. R. K., Jain, R. K.: Numerical Methods, Problems and solutions, Wiley Eastern Ltd., New Age International Publishers Ltd., 1995 Edition.
4. Kandasamy, P., Thilagavathy, K., and Gunavathy, K.: Numerical Methods, S. Chand & Co. Ltd., New Delhi, 2003 Edition.

**Computer programming in C & Computer Oriented Numerical Analysis
(Practical)**

Part: (B)

(Number of Practice/Teaching hours: 48; Marks : 40)

Part-1

The following programs are to be practiced.

1. Roots of quadratic equation $Ax^2+Bx+C=0$,
2. Arrangement a given set of numbers in increasing/decreasing order; calculation of mean,
3. Evaluation e^x , $\sin x$, $\cos x$, $\log(1+x)$ using power series method,
4. Addition, subtraction and multiplication of matrices using function,
5. Evaluation of factorial of a positive integer and evaluation of binomial coefficients,
6. Determination of the transpose, determinant of the given matrix.(up to order 4),
7. Determination of the inverse of a given real matrix (up to order 4),
8. Searching a pattern in a given text and replacing every occurrence of it with another given string,
9. Writing a given number in words using function,
10. Copying the contents of one text to another text file using command line arguments,
11. Merging two text files to another text file,
12. Copying the contents of one text file to any number of given files using command line arguments,
13. Printing of every line of a text file containing a given pattern.

Part – 2

1. Lagrange's Interpolating Polynomial.
2. Newton Forward Difference Interpolating Polynomial.
3. Newton Backward Difference Interpolating Polynomial.
4. Simpson's 1/3 rule for Numerical Integration.
5. Trapezoidal Rule Rule for Numerical Integration.
6. Newton Raphson Method.
7. Regula-Falsi method.
8. Bisection method.
9. Gauss Elimination method.
10. Heun's method for solving an Initial Value problem.

NOTE : There will be practical examination for 40 marks of three hours duration of which 15 marks will be for Part 1 and 25 marks for Part 2 . All output should be in the form of an output file.

**HOPT 62 : OP-2
OPERATIONS RESEARCH**

(Number of Teaching hours: 80; Time: 3 hrs; Marks: 100)

(To answer five questions, choosing one out of two questions from each unit)

UNIT I. Linear programming problem, mathematical formulation of linear programming problem, feasible solution, solution space, linear function on convex set, graphical method of solution (including exceptional cases).

UNIT II. Standard and canonical form of LPP, duality in linear programming problem; basic feasible solution, optimal solution, slack and surplus variables, initial simplex table, terminal simplex table, pivot entry, algorithm of simplex method.

UNIT III. Simplex method of solution of LPP; theory of games, two-person-zero-sum games, the maximin–minimax principle, fair and strictly determinable game, saddle point, rule for determining a saddle point.

UNIT IV. Relation between minimax and maximin game without saddle point, pure and mixed strategies, dominance property; modified dominance property, reduction of a game problem to a linear programming problem, and its solution, graphical solution of $2 \times n$ and $m \times 2$ games.

UNIT V : Markov analysis; probability vectors, stochastic and regular stochastic matrices; brand switching analysis; fixed points of square matrices; relationships between fixed points and regular stochastic matrices; Markov chains; higher transition probabilities; stationary distribution of regular Markov chains; absorbing states.

BOOKS

Text Book :

1. Swarup, K., Gupta, P. K. and Singh, M. M. : Operations Research, Sultan Chand & Sons, New Delhi, 2002 Edition.

Reference Books:

1. Gupta, P. K., and Hira, D. S. : Operations Research – An Introduction, S. Chand & Co. Ltd., New Delhi, 2002 Edition.

2. Rao, S. S. : Optimisation Theory and Applications, Wiley Eastern Ltd., New Delhi, 2001 Edition.

3. Malik, T. N. : Linear Programming, U. N. Dhar & Sons Publications, Kolkata, 2001 Edition.

HOPT 62 : OP 3
Hydro Mechanics

(100 marks, 80 lectures)

(To answer five questions, choosing one out of two questions from each unit)

Unit: I Lagrangian and Eulerian methods of describing fluid motion; velocity of a fluid particle; material, local and convective derivatives; acceleration of a fluid particle; streamline motion and turbulent motion; steady flow; streamlines and path lines; velocity potential; vorticity vector; flux of a fluid; equation of continuity by Euler's method; equation of continuity by Lagrange's method; equivalence of the Eulerian and Lagrangian forms of the equation of continuity; equation of continuity in cartesian, spherical polar and cylindrical coordinates; certain symmetrical forms of the equation of continuity; boundary surface.

Unit : II Euler's equations of motion (vector and cartesian forms); conservative field of force; pressure equation; Bernoulli's theorem-case of no velocity potential; Lagrange's equations of motion; equations of motion under impulsive forces (vector and cartesian forms); energy equation.

Unit : III Two dimensional motion; stream function or current function; properties of the stream function; irrotational motion in two dimensions; complex potential and velocity; Cauchy-Riemann equations in polar form; images; sources and sinks in two dimensions; two dimensional doublet; complex potential for a doublet; images of a simple source w.r.t. two dimensional motion; image of a source w.r.t. a circle; motion of fluid symmetrical about an axis- Stoke's stream function, determination of ψ ; image of a source w.r.t. a sphere; image of a doublet.w.r.t. a sphere; Blasius theorem.

Unit : IV Perfect fluid; fluid pressure, pressure at a point in a fluid in equilibrium, pressure of heavy fluids; transmissibility of fluid pressure, conditions for equilibrium; pressure equation, surfaces of equal pressure; surfaces of equi-density; floating bodies-conditions of equilibrium of a floating body, freely floating bodies and bodies floating under constraint; stability of floating bodies.

Unit : V Resultant thrust on a solid wholly or partially immersed in a heavy fluid at rest, resultant vertical thrust on a surface exposed to the pressure of a heavy fluid, resultant pressure on surfaces (plane or curved)- horizontal thrust and vertical thrust, centre of pressure; gases; internal energy of a gas, reversible isothermal change, reversible adiabatic change, atmosphere; equilibrium of an isothermal atmosphere, convective equilibrium.

BOOKS

Text Books :

1. Verma, B.G. & Gupta, K.P. : Hydro-dynamics, Pragati Prakashan, Meerut (U.P.), 2003 Edition.
2. Kar, J.M. : Hydrostatics, K.P. Basu Publishing Company, 1992 Edition.

Reference books:

1. Jha, D..K. : Text Book of Hydrodynamics, Discovery Publishing House Pvt. Ltd, 2005 Edition.
2. Ray, M. & Sharma, H.S. : Hydrostatics, S. Chand Publications, 1999 Edition.

HOPT 62 : OP 4

Financial Mathematics

(100 marks, 80 lectures)

(To answer five questions, choosing one out of two questions from each unit)

Unit-I: Basic Principles, Arbitrage and risk aversion, interest (simple and compound, discrete and continuous), time value of money, inflation, net present value, internal rate of return (calculation by bisection and Newton-Raphson methods), Comparison of NPV and IRR .

Unit-II: Concept of Bonds, bond prices and yields, Macaulay and modified duration, term structure of interest rates: spot and forward rates, explanations of term structure, running present value, floating- rate bonds, immunization, convexity, putable and callable bonds . Marks:20

Unit -III: Asset return, short selling, portfolio return, (brief introduction to expectation, variance, covariance and correlation) , random returns, portfolio mean return and variance, diversification, portfolio diagram, feasible set, Markowitz model, Two fund theorem, Capital market line, Capital Asset Pricing Model, Use of CAPM in investment analysis and as a pricing formula, Jensen's index .

Unit-IV: Forwards and futures, marking to market, currency futures, hedging (short, long, cross, rolling), optimal hedge ratio, hedging with stock index futures, Lognormal distribution, Lognormal model, Geometric Brownian motion for stock prices, Binomial tree model for stock prices, parameter estimation.

Unit-V: Insurance Fundamentals- Insurance defined, Meaning of loss, Chances of loss , peril, hazard and proximate cause in insurance, Costs and benefits of insurance to the society and branches of insurance- life insurance and various types of general insurance , Life insurance mathematics- Construction of mortality tables, Computation of premium of life- insurance for a fixed duration and for the whole life, Determination of claims for general insurance- using Poisson Distribution and Negative Binomial Distribution- the Polya Case .

BOOKS

Text books:

1. David G. Luenberger, Investment Science, Oxford University Press, Delhi, 1998
2. John C. Hull, Options, Futures and Other Derivatives (6 Edition), Prentice – Hall India, 2006
3. Sheldon M. Ross, An Elementary Introduction to Mathematical Finance, (2nd Edition), Cambridge University Press, USA, 2003.
4. Sankalp[Srivastava, Financial Mathematics , New Age International, (paper back). 2011.
5. Samir Kumar Chakraborty, Financial Mathematics, New Age international (Paper back), 2011.

Reference books:

1. Aswath Damodaran, Corporate Finance- Theory and Practice, John Wiley and Sons, Inc
2. Mark S. Dorfman, Introduction to Risk Management and Insurance, Prentice Hall, Englewood Cliffs, New Jersey,
3. C. D. Daykin, T. Pentikainen and M. Pesonen, Practical Risk Theory for Actuaries, Chapman and Hall .

OP 5

Discrete Mathematics

(100 marks, 80 lectures)

(To answer five questions, choosing one out of two questions from each unit)

Unit-I : Mathematical induction, Principle of inclusion and exclusion, Pigeon hole principle, Finite combinatorics, Generating functions, Partitions, Recurrence relations and Recursive Algorithms, Linear Recurrence relations with constant coefficients, Homogeneous solutions, Total Solution, Solution by the method of generating functions.

Unit: II Definition, examples and basic properties of ordered sets, duality principle, lattices as ordered sets, lattices as algebraic structures, sublattices, products and homomorphisms.

Unit-III: Definition, examples and properties of modular and distributive lattices , Boolean algebras, Boolean polynomials, minimal forms of Boolean polynomials, Quinn – McCluskey method, Karnaugh diagrams, switching – circuits and application of switching circuits .

Unit- IV: Definition, examples and basic properties of graphs, complete graphs, bi-partite graphs, Paths and Circuits, Hamiltonian paths, Shortest paths, Eulerian paths, Traveling salesman problem.

Unit-V : Block, Cut Points, Bridges, Block graphs, Cut point graphs, Trees , Characterization of trees, Connectivity and Line Connectivity, Graphical variation of Menger’s theorem .

BOOKS

Text Books:

1. C.L. Liu, Elements of Discrete Mathematics, (Second Edition) , McGraw Hill, International Edition.
2. M. K. Sen, Introduction to Discrete Mathematics, Allied Publishers
3. B. A. Davey and H. A. Priestley, Introduction to Lattices and Order, Cambridge University Press, Cambridge
4. Edgar G. Goodaire and Michael M. Parmenter, Discrete mathematics with Graph theory (2nd Edition), Pearson Education Pvt Ltd, Indian Reprint 2003 .
5. S.K. Sankar, A Text book of Discrete Mathematics, Published by S. Chand, 2008.
6. Harish Mittal and Vinay Kr. Goel , A Text Book of Discrete mathematics, , published by I.K. International, 2010. .

Reference Books:

1. Michael Towusend, Discrete mathematics, Applied Mathematics and Graph theory
2. J.P. Tremblay and R. Manohar, Discrete Mathematics Structures with Applications to Computer Science, McGraw- Hill Book Co.
3. K.R. Parthasarathi, Basic Graph theory
4. S. Lipschutz, ;Marc Lipson Schaum's outlines of Discrete Mathematics , 2007.

OP 6

Mathematical Modeling

(100 marks, 80 lectures)

(To answer five questions, choosing one out of two questions from each unit)

Unit- I: Introduction, Basic Steps of Mathematical Modeling, its needs, types of models, limitations, Elementary ideas of dynamical systems, autonomous dynamical systems in the plane- linear theory, Equilibrium point, node, saddle point, focus, centre and limit-cycle idea with simple illustrations and figures, Linearization of non-linear plane autonomous systems.

Unit-II: Population Models: Basic concepts, Exponential growth model, formulation , solution, interpretation and limitations. Compensation and depensation, Logistic growth model, formulation, solution, interpretation and limitations. Lotka- Volterra model of two competing species, formulation, solution, interpretation and limitations.

Unit-II: Epidemic Models: Basic concepts, Simple epidemic model, formulation, solution, interpretation , and limitation, General epidemic model, formulation, solution, interpretation and limitations .

Unit-IV: Economic models: Production and supply functions, price-elasticities, utility of consumption and consumer surplus, pure competition, competitive equilibrium , monopoly versus competition, duopoly, oligopoly, conjectural variation, theory of production, production function , Cobb- Douglas production function and its properties , Costs of production and related models .

Unit-V: Mathematical modeling in Bio-logical Environment: Blood flow and oxygen transfer , Modeling blood flow, viscosity, Poiseuille law, mathematical formulation of the problem, solution and interpretation, oxygen transfer in red cells, mathematical formulation, solution, interpretation and limitations.

BOOKS

Text Books:

1. Mark M. Meerschaert, Mathematical Modeling, Academic Press, New York, 1993
2. W. Meyer , Concepts of Mathematical Modeling, McGraw Hill, New York, 1994
3. E. Beltrami , Mathematics for Dynamic Modeling, Academic Press, Orlando, Florida, 1987

Reference books:

4. N. Bailey, The Mathematical Theory of Infectious Diseases, Hafner press, New York, 1975

5. Mathematical modeling in the biological environment, MTE- 14, Indira Gandhi National Open University, New Delhi 1998
6. J.N. Kapur, Insight into mathematical modeling, Indian National Science Academy, New Delhi 1983
7. M. Braun, Differential Equations , ad their Applications, Springer, New York, 1980